X-921-74-144
PREPRINT

NASA TM X- 70676

THE ROAD PROGRAM

C. A. WAGNER
B. C. DOUGLAS
R. G. WILLIAMSON

JANUARY 1974

GSFC
GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

43

For information concerning availability
of this document contact:

Technical Information Division, Code 250
Goddard Space Flight Center
Greenbelt, Maryland 20771

(Telephone 301-982-4488)

THE ROAD PROGRAM

C. A. Wagner
Earth Survey Applications Division
Geodynamics Branch

B. C. Douglas
R. G. Williamson
Wolf Research and Development Corp.
Riverdale, Md. 20840

January 1974

Goddard Space Flight Center
Greenbelt, Maryland

ABSTRACT

The philosophy, history, operation, calibration of and some analyses with the ROAD (Rapid Orbit Analysis and Determination) program are described. This semi-numeric trajectory program integrates and analyses mean element variations for earth orbits with great efficiency. Through it's use, extensive zonal, resonant harmonic and earth tidal determinations have been made at Goddard Space Flight Center since 1969.

CONTENTS

	Page
INTRODUCTION	1
HISTORY	3
THE ROAD INTEGRATOR	4
Geopotential Effects	4
Third Body Gravity Effects (Direct)	9
Drag Perturbations	9
Radiation Pressure Effects	15
Precession and Nutation	17
Solar and Lunar Tides	18
Element Secular Rates and Accelerations	20
ORBIT AND GEODETIC PARAMETER ESTIMATION	21
RESULTS AND CALIBRATIONS	22
SUMMARY	26
REFERENCES	28

THE ROAD PROGRAM

INTRODUCTION

At Goddard Space Flight Center, the most general computational system for analyzing long term gravitational and non-gravitational effects on satellite orbits has been the ROAD (Rapid Orbit Analysis and Determination) program. This program is a multiple arc/satellite orbit generator which can also estimate (from real data) a wide variety of the influential geodetic parameters. It uses mean Kepler elements as the usual data type.

The ROAD orbit generator generally integrates numerically, orbit-averaged Kepler element variations determined from a number of sources. The variations due to the geopotential (to 40, 40), are determined from the right hand side of the Lagrange planetary equations in its standard form. Only those geopotential disturbances are usually integrated which have long term effects on the orbits, permitting time steps of the order of a day or more and extremely fast ephemerides generation. Direct luni-solar gravity and the indirect luni-solar tidal effects on the orbits are evaluated from a similar disturbing potential which selects only long period or orbit averaged terms for integration. Radiation pressure variations are evaluated from a quasi-potential for full sunlight conditions. When the orbit is partially in shadow, an orbit-averaged force is evaluated by averaging the disturbance from shadow exit to shadow entrance.

Precession and nutation effects of the movement of the earth's polar axis are accounted for by integrating in a true of epoch inertial system. All forces are initially calculated in true of date coordinates and then rotated to true of epoch (using a precession-nutation rotation matrix). After integration, the true of epoch elements are then rotated back to true of date for comparison with real data which are usually given in these (latter) coordinates.

Drag is the only perturbation not evaluated by a potential or quasi-potential. Therefore its element variations are not evaluated from the right hand side of the standard planetary equations. Instead the gaussian form of the variation equations are employed with the normal, circumferential and radial components of the drag force evaluated from a one revolution J_2 perturbed trajectory through a model atmosphere.

In its estimation mode, ROAD has the capability of solving for the following arc dependent parameters: The six initial Kepler elements, an offset or bias in the semi-major axis observations, up to 5 element rates and accelerations for each of the initial Kepler elements, and a drag and radiation pressure coefficient for each arc. In addition, it can solve for the following geodetic parameters, common to all the satellite arcs observed: The earth's gaussian gravity constant and earth radius, geopotential harmonics to (40, 40), and earth tidal harmonics (love numbers) and their associated lag angles to 4th degree. ROAD determines these parameters by a Baysian least squares process using the Kepler element data as "observables." The partials of these observables with respect to the parameters are generally found by numerical integration of variation equations.

HISTORY

The orbit generator of ROAD, and the basic idea of integrating mean or slowly varying Kepler elements, originated with the (resonant orbit) RESORB program developed by B. C. Douglas and G. S. Gedeon at TRW, Inc. in 1966 [Gedeon, Douglas and Palmiter, 1967]. The original purpose of this program was to study the complex evolution of deeply resonant orbits for which no analytic theory exists. Later RESORB, with only fixed geopotential and third body forces, was extended to include drag and became (in 1967) the satellite lifetime (Rapid Orbit Prediction) Program, ROPP [Wexler and Gedeon, 1967], used extensively at Goddard Space Flight Center. Still later in 1968, ROPP was modified at Goddard Space Flight Center to give it a limited estimation capability (using secant partials) for state and geopotential recovery from long arcs of mean Kepler elements. In addition, the effects of radiation pressure were added, using Kaula's quasi-potential [Kaula, 1962], to account for significant changes in highly eccentric resonant orbits.

The results of resonant geopotential determinations with this initial version of the ROAD (Rapid Orbit Analysis and Determination) program were published in Wagner, 1969, Wagner and Douglas, 1969 and Wagner, 1970b. In 1970, the long term rates due to the interaction of short period terms in the earth's oblateness were added to the ROAD integrator so that a proper analysis of the evolution of close earth satellites could be made. Using this version of ROAD, preliminary results for zonal recovery from 2 satellites were published in Wagner, Putney and

Fisher, 1970. At the same time a new program was written using rigorous numerical integration of variation equations for partials and a complete Baysian (a priori) least squares scheme for parameter estimation [Williamson, 1970, Guion, Lynn and Lynn, 1970 and Douglas, Dunn and Williamson, 1972]. Results from this new ROAD program were published in 1970 and 1972 on the determination of both resonant and zonal geopotential terms from a large number of satellite orbits [Wagner, 1970a, Wagner, 1972].

THE ROAD INTEGRATOR

An Adams multistep integration scheme is used to solve the first order system of Lagrange's equations describing the satellite's motion:

$$\frac{d\,s_i}{d\,t} = S_i\ (s,\ p,\ t),\ s_i\ (t = 0) = s_i\ (0)_{\text{Given}};\ i = 1,\ 2, \ldots 6, \tag{1}$$

where the s are the satellite orbit's 6 state variables; a, semi-major axis; e, eccentricity; I, inclination; w, argument of perigee; N, right ascension of the ascending node and M, the mean anomaly, and the p are geodetic parameters describing the force model.

Geopotential Effects

The rates S_i for geopotential effects are derived from the standard form [Kaula, 1966] of (1):

$$\frac{d\,a}{d\,t} = \frac{2}{n\,a}\frac{\partial R}{\partial M}$$

$$\frac{de}{dt} = \frac{(1-e^2)}{n a^2 e} \frac{\partial R}{\partial M} - \frac{(1-e^2)^{1/2}}{n a^2 e} \frac{\partial R}{\partial w}$$

$$\frac{dI}{dt} = \frac{\cot I}{n a^2 (1-e^2)^{1/2}} \frac{\partial R}{\partial w} - \frac{\csc I}{n a^2 (1-e^2)^{1/2}} \frac{\partial R}{\partial N}$$

$$\frac{dw}{dt} = \frac{-\cot I}{n a^2 (1-e^2)^{1/2}} \frac{\partial R}{\partial I} + \frac{(1-e^2)^{1/2}}{n a^2 e} \frac{\partial R}{\partial e}$$

$$\frac{dN}{dt} = \frac{\csc I}{n a^2 (1-e^2)^{1/2}} \frac{\partial R}{\partial I}, \tag{2}$$

$$\frac{dM}{dt} = n - \frac{(1-e^2)}{n a^2 e} \frac{\partial R}{\partial e} - \frac{2}{na} \frac{\partial R}{\partial a},$$

where $n = (\mu / a^3)^{1/2}$; n being the satellite's mean motion and μ the earth's gaussian gravity constant. R is the disturbing (non central) potential which, for the earth's gravitational field (fixed part as distinguished from time varying), has been expressed by Kaula [Kaula, 1966] in terms of the above Kepler elements as:

$$R_e = \sum_{\ell=2}^{\infty} \sum_{m=0}^{\ell} \sum_{p=0}^{\ell} \sum_{q=-\infty}^{\infty} \frac{\mu a_e^{\ell}}{a^{\ell+1}} J_{\ell m} F_{\ell m p}(I) G_{\ell p q}(e) S_{\ell m p q}, \tag{3}$$

where

$$S_{\ell mpq} = \begin{Bmatrix} \cos \\ \\ \sin \end{Bmatrix}_{\ell-m \text{ odd}}^{\ell-m \text{ even}} (\ell - 2p) w + (\ell - 2p + q) M + m (N - \theta_e - \lambda_{\ell m}),$$

and a_e is the earth's equatorial radius, θ_e is the Greenwich sidereal time, and $F_{\ell mp}$ (I) and $G_{\ell pq}$ (e) are polynomials depending on inclination and eccentricity alone, respectively. The $J_{\ell m}$ and $\lambda_{\ell m}$ are the amplitudes and phase angles of the spherical harmonics of the potential, related to the ordinary C&S coefficients actually estimated in ROAD by

$$J_{\ell m} = + (C^2_{\ell m} + S^2_{\ell m})^{1/2}$$

$$m \lambda_{\ell m} = \mathrm{Tan}^{-1} (S_{\ell m}/C_{\ell m}).$$

The standard form of R_e in terms of spherical harmonics is:

$$R_e = \frac{\mu}{r} \sum_{\ell=2}^{\infty} \sum_{m=0}^{\ell} \left(\frac{a_e}{r}\right)^{\ell} P_\ell^m (\sin \phi) \{C_{\ell m} \cos m \lambda + S_{\ell m} \sin m \lambda\}, \qquad (4)$$

where r, ϕ and λ are the satellite's distance from the center of mass of the earth, geocentric latitude and longitude; and the P_ℓ^m are the associated Legendre polynomials.

The efficient functioning of the ROAD program depends upon the removal of all short periodic effects from the equations of motion. For the geopotential, this is accomplished by integrating only those terms (ℓ, m, p, q) for which the argument of $S_{\ell mpq}$ is slowly varying. These long period terms fall into two classes:

For zonal harmonics (m = 0); $\ell - 2p + q = 0$, and for longitude dependent (resonant) harmonics ($m \neq 0$), $\ell - 2p + q = m/s$ are the indicial equations which must be satisfied. For the resonant case, s here is a rational number close to the actual mean motion (n) of the satellite in revolutions per day. The so called beat period of the resonant orbit (with respect to the s commensurability) is the period it takes for the dominant argument of S to go through 360° when $\ell - 2p + q = m/s$. For near circular orbits this is generally the term for which $q = 0$.

This scheme is equivalent to integrating mean, or orbit averaged Kepler elements and results in the proper interaction of all long-periodic effects with each other. However, interactions of short-periodic terms ($\ell - 2p + q \neq m/s$, for $m \neq 0$ and $\ell - 2p + q \neq 0$, for $m = 0$) with each other and with long-periodic terms are lost. But the only case where this neglect is significant are the long periodic effects caused by the mutual interaction of the short periodic effects due to the earth's oblateness ($J_{2,0}$). When only long period terms are included, the ROAD integrator adds the following element rates derived from the second order oblateness potential due to Brouwer [Brouwer, 1959].

$$\frac{d\,a}{d\,t} = 0$$

$$\frac{d\,e}{d\,t} = \left[\frac{1}{e\left(\frac{L}{G}\right)L}\right]\left\{(2\sin 2w)\,L\,n_0\,\gamma_2^2\left[\frac{-3}{16}\left(\frac{L^5}{G^5} - \frac{L^7}{G^7}\right)\right.\right.$$

$$\left.\left.\cdot\left(1 - \frac{16\,H^2}{G^2} + \frac{15\,H^4}{G^4}\right)\right]\right\}$$

$$\frac{d\,I}{d\,t} = \frac{2\,(H/G)^2}{G\sin 2I}\left\{(2\sin 2w)\,L\,n_0\,\gamma_2^2\left[\frac{-3}{16}\left(\frac{L^5}{G^5} - \frac{L^7}{G^7}\right)\right.\right.$$

$$\left.\left.\cdot\left(1 - \frac{16\,H^2}{G^2} + \frac{15\,H^4}{G^4}\right)\right]\right\}$$

$$\frac{dw}{dt} = n_0 \gamma_2^2 \left[\frac{75}{32} \frac{L^6}{G^6} + \frac{9}{4} \frac{L^7}{G^7} - \frac{105}{32} \frac{L^8}{G^8} + \left(\frac{-189}{16} \frac{L^6}{G^6} - \frac{18 L^7}{G^7} \right. \right.$$

$$\left. \left. + \frac{135}{16} \frac{L^8}{G^8} \right) \frac{H^2}{G^2} + \left(\frac{135}{32} \frac{L^6}{G^6} + \frac{135}{4} \frac{L^7}{G^7} + \frac{1155}{32} \frac{L^8}{G^8} \right) \frac{H^4}{G^4} \right]$$

$$+ \frac{3 \mu^6 k_2^2}{16 L^{10}} \left\{ \left(\frac{L^5}{G^5} - \frac{L^7}{G^7} \right) \left(\frac{32 H^3}{G^3} - \frac{60 H^5}{G^5} \right) \left(\frac{1}{H} \right) \right.$$

$$\left. + \left(1 - \frac{16 H^2}{G^2} + \frac{15 H^4}{G^4} \right) \left(\frac{-5 L^6}{G^6} + \frac{7 L^8}{G^8} \right) \left(\frac{1}{L} \right) \right\} \cos 2w$$

$$\frac{dN}{dt} = n_0 \gamma_2^2 \left[\left(\frac{27}{8} \frac{L^6}{G^6} + \frac{9}{2} \frac{L^7}{G^7} - \frac{15}{8} \frac{L^8}{G^8} \right) \frac{H}{G} + \left(\frac{-15 L^6}{8 G^6} - \frac{27 L^7}{2 G^7} \right. \right.$$

$$\left. \left. - \frac{105 L^8}{8 G^8} \right) \frac{H^3}{G^3} \right] + \frac{3 \mu^6 k_2^2}{16 L^{10}} \left[\left(\frac{L^5}{G^5} - \frac{L^7}{G^7} \right) \left(\frac{-32 H^2}{G^2} + \frac{60 H^4}{G^4} \right) \right.$$

$$\left. \cdot \left(\frac{1}{H} \right) \right] \cos 2w$$

$$\frac{dM}{dt} = n_0 \gamma_2^2 \left[\frac{75 L^5}{32 G^5} + \frac{3 L^6}{2 G^6} - \frac{45 L^7}{32 G^7} + \left(- \frac{135 L^5}{16 G^5} - \frac{9 L^6}{G^6} + \frac{45 L^7}{16 G^7} \right) \right.$$

$$\left. \cdot \left(\frac{H^2}{G^2} \right) + \left(\frac{75 L^5}{32 G^5} + \frac{27 L^6}{2 G^6} + \frac{315 L^7}{32 G^7} \right) \frac{H^4}{G^4} \right]$$

$$+ \frac{3 \mu^6 k_2^2}{16 L^{11}} \left[\left(1 - \frac{16 H^2}{G^2} + \frac{15 H^4}{G^4} \right) \left(\frac{-5 L^5}{G^5} + \frac{3 L^7}{G^7} \right) \right] \cos 2w, \qquad (5)$$

where the Delaunay variables are: $L = (\mu a)^{1/2}$, $G = L (1 - e^2)^{1/2}$ and $H = G \cos I$; and $n_0 = \mu^2/L^3$, $\gamma_2 = \mu^2 k_2/L^4$, $k_2 = J_2 (a_e)^2/2$, and $J_2 = -J_{2,0}$.

Third Body Gravity Effects (Direct)

For the 3rd body perturbations, an entirely analogous development has been made in Kaula 1962, and is used in ROAD. In this case, the perturbations can be sorted out according to frequency in much the same manner as the geopotential perturbations. The long periodic terms of the 3rd body disturbing function are the only ones presently coded in ROAD and have the form

$$\overline{R}_n = f\,(a,\ e,\ I,\ a^*,\ e^*,\ I^*)\ \cos\,[(n-2\,h)\,w^* - (n-2\,p)\,w + (n-2\,h+j)\,M^*$$

$$+\,m\,(N-N^*)],$$

where n here is the degree of the third body potential ($2 \le n \le 4$, as coded, and where starred quantities refer to orbit elements of the disturbing body. Analogous to the geopotential case; $0 \le h \le n$, $0 \le p \le n$ and $0 \le m \le n$. Since f is proportional to $(e^*)^{|j|}$, only $-4 \le j \le 4$ is coded and can be limited further on option. A further option is to ignore the less than monthly lunar terms where $n - 2h + j \neq 0$.

Drag Perturbations

The method of computation of the long periodic and secular variations of the elements due to atmospheric drag has undergone extensive development in ROAD. Originally, analytic expressions similar to those given in King-Hele, 1964 were used, but when these expressions were modified to include non-linearity of scale height and time dependence, the complexity became very great, and separate developments for low and high eccentricities were required

[Wexler and Gedeon, 1967]. Starting in 1971 this method was abandoned in favor of a simple, purely numerical scheme (B. Chovitz, personal communication) originally including only the effects on a and e.

The integrals that give the average time rates-of-change of the elements due to drag are evaluated numerically by a 9-point Gauss quadrature integration. At each integration point it is necessary only to compute the altitude of the satellite, and evaluate the density of the atmosphere from any atmosphere model. The ROAD program currently uses either the Jacchia 1968 or 1971 atmosphere models [Diamante and Der, 1972].

The element rates are computed using the Gaussian form of the equations of motion, which is expressed in terms of radial (F_r), transverse (F_t), and normal forces (F_n). The drag force is the usual simple model:

$$\bar{F}_d = \frac{1}{2} C_D \frac{A}{m} \rho \, v_r \, \bar{v}_r$$

where

C_D is the satellite drag coefficient

A is the projected cross-sectional area of the satellite normal to $\bar{v}_r$

m is the mass of the satellite

ρ is the density of the atmosphere at the satellite position, and

$\bar{v}_r$ is the velocity vector of the satellite relative to the atmosphere.

The most significant problem in computing the correct drag is the calculation of an accurate spheroid height of the satellite. This is done on the osculating J_2 perturbed orbit over which the drag is averaged. The formula used was developed in Kozai, 1959 and modified to use the mean semi-major axis instead of his "geometric" semi-major axis. That is:

$$\Delta r = \frac{J_2 a_e^2}{2p}\left(1 - \frac{3}{2}\sin^2 I\right)$$

$$\left[1 + \frac{1}{e}\left(1 - \sqrt{1-e^2}\right)\cos f + \frac{2r}{a\sqrt{1-e^2}}\right]$$

$$+ \frac{J_2 a_e^2}{4p}\sin^2 I \left[\cos 2(w+f)\right.$$

$$\left. - \frac{9}{4}\frac{e}{1-e^2}\sin f \sin 2w\right],$$

where $p = a(1 - e^2)$ and f is the true anomaly.

Thus the satellite's radius vector has magnitude $r + \Delta r$ where r is computed from the two-body formula.

The spheroid height is then obtained by subtracting the spheroid radius of the Earth at the latitude under consideration:

$$R_{Earth} = a_e \left[1 - \left(f + \frac{3}{2}f^2\right)\sin^2\phi' + \frac{3}{2}f^2\sin^4\phi'\right].$$

where in this particular formula,

f is the flattening of the Earth, and

ϕ' is the satellite's geocentric latitude.

The atmosphere is assumed to rotate with the Earth. The rotation rate of the Earth resolves into a component ($\dot{\theta}_e \cos I$) normal to the orbit and a component ($\dot{\theta}_e \sin I$) perpendicular to the angular momentum vector, $\bar{h}$, of the satellite, which is perpendicular to the vector, $\overline{N}$, pointing toward the ascending node.

The satellite's velocity vector is given by

$$\dot{\bar{r}} = (\dot{r},\ r\,\dot{f},\ 0)$$

in radial, transverse (or circumferential), and normal components, where $\bar{r}$ is the satellite's position vector.

The relative velocity vector is then

$$\bar{v}_r = [\dot{r},\ r\,\dot{f} - \dot{\theta}_e\, r \cos I,\ \dot{\theta}_e\, r \sin I \cos (w + f)]\,.$$

From the vis viva or energy integral,

$$\dot{\bar{r}} \cdot \dot{\bar{r}} = \dot{r}^2 + r^2\, \dot{f}^2 = \mu \left(\frac{2}{r} - \frac{1}{a}\right).$$

Also, the angular momentum h is given by

$$h = r^2\, \dot{f} = \sqrt{\mu\, a\, (1 - e^2)}\,.$$

Hence:

$$v_r^2 = \mu \left(\frac{2}{r} - \frac{1}{a}\right) - \frac{2\,h}{r}\, \dot{\theta}_e \cos I$$

$$+ \dot{\theta}_e\, r^2\, [\cos^2 I + \sin^2 I \cos^2 (w + f)]\,.$$

This last formula is used to compute the magnitude of the relative velocity vector.

The Gaussian form of the variation equations for the perturbations of the Kepler elements are:

$$\frac{da}{dt} = \frac{2a^2}{h}[F_r \, e \sin f + F_t (1 + e \cos f)]$$

$$\frac{de}{dt} = \frac{h}{\mu}\left[F_r \sin f + F_t \left(\cos f + \frac{\cos f + e}{1 + e \cos f}\right)\right]$$

$$\frac{dI}{dt} = \frac{1}{h} r F_n \cos (w + f)$$

$$\frac{dN}{dt} = \frac{1}{h} \frac{r F_n}{\sin I} \sin (w + f)$$

$$\frac{dw}{dt} = \frac{h}{e\mu}\left(- F_r \cos f + F_t \left(1 + \frac{r}{p}\right) \sin f\right) - \frac{dN}{dt} \cos I$$

$$\frac{dM}{dt} = \frac{\mu (1 - e^2)^{1/2}}{h a} - \left(\frac{dw}{dt} + \frac{dN}{dt} \cos I\right) (1 - e^2)^{1/2} - \frac{2r}{h} F_r (1 - e^2)^{1/2}.$$

For the drag case being considered:

$$F_r = - F_d \dot{r}$$

$$F_t = - F_d (r \dot{f} - \dot{\theta}_e r \cos I)$$

$$F_n = - F_d (\dot{\theta}_e r \sin I \cos (w + f)$$

Using the usual two body formulae for relating Kepler variables, the element rates are simplified to:

$$\frac{d\,a}{d\,t} = -\,2\,a^2\,F_d\left(\frac{1 + e^2 + 2\,e\cos f}{p} - \frac{p\,\dot{\theta}_e\cos I}{h}\right)$$

$$\frac{d\,e}{d\,t} = -\,F_d\left[2\,e + 2\cos f - \frac{\dot{\theta}_e\,r^2\cos I}{h}(e\cos^2 f + 2\cos f + e)\right]$$

$$\frac{d\,I}{d\,t} = -\frac{F_d}{h}\cdot\left[r^2\,\dot{\theta}_e\sin(I)\cos^2(w + f)\right]$$

$$\frac{d\,N}{d\,t} = \frac{-\,F_d}{h}\,r^2\,\dot{\theta}_e\sin(w + f)\cos(w + f)$$

$$\frac{d\,w}{d\,t} = \frac{-\,F_d}{e}\left[2 - \dot{\theta}_e\,\frac{r^2\cos I}{h}(2 + e\cos f)\right]\sin f - \frac{d\,N}{d\,t}\cos I$$

$$\frac{d\,M}{d\,t} = (1 - e^2)^{1/2}\left[\frac{\mu}{h\,a} - \frac{d\,w}{d\,t} - \frac{d\,N}{d\,t}\cos I + \frac{2\,e\,F_d\,r}{p}\sin f\right].$$

These equations are now averaged over one revolution in True Anomaly to obtain the mean element rates which are added (on option) to the other rates used in the ROAD integrator. This averaging is done numerically by a 9 point Gauss quadrature method with the middle point placed at perigee to insure an accurate computation of the maximum drag.

Radiation Pressure Effects

The scheme used to calculate the long-periodic radiation pressure effects is that given in Kaula, 1962. He notes that for a satellite entirely in sunlight, a quasi-potential (for the orbit averaged disturbance) can be developed for radiation pressure. Kaula's quasi-potential is used as is in the Lagrange Planetary equations for sunlight conditions. However, if a shadow is present, an integration of the derivatives of the full potential from exit point to entry point is made to obtain the orbit-averaged disturbance. The values of the eccentric anomaly at these points are obtained by solution of the quartic equation given in Kaula's paper. The satellite is assumed to be spherical, and the earth's shadow cylindrical. The radiation force f_r (actually, the acceleration) is given as (I/C) (A/M) C_R where I is the Solar Flux in space (1.37×10^6 erg/cm^2-sec) and C is the velocity of light. The original subroutine for these effects in ROAD was supplied by Bernard Chovitz [B. Chovitz, Personnal Communication, 1970].

For a shadowless orbit, the long-term (orbit averaged) potential is as given in Kaula, 1962:

$$
\begin{aligned}
\overline{R} = - f_r\,(3\,a\,e/2)\,[&\cos^2(I/2)\sin^2(\epsilon/2)\cos(w+N+\lambda_\odot) \\
&+\cos^2(I/2)\cos^2(\epsilon/2)\cos(w+N-\lambda_\odot) \\
&+\sin^2(I/2)\cos^2(\epsilon/2)\cos(w-N+\lambda_\odot) \\
&-1/2\sin I\sin\epsilon\cos(w+\lambda_\odot)+1/2\sin I\sin\epsilon\cos(w-\lambda_\odot)],
\end{aligned}
$$

where ϵ and $\lambda_\odot$ are the inclination of the ecliptic (23.45°) and the sun's true longitude.

The derivatives of the quasi-potential with respect to the orbit elements are then used in the standard form of the Lagrange equations, (2), to form the element rates from the radiation pressure for the shadowless orbit.

For the shadowed orbit, the long term, orbit averaged, derivatives are given in terms of the exit and entry eccentric anomalies, E_0 and E_1, as:

$$\frac{\partial R}{\partial a} = \frac{f_r}{2\pi}\Bigg[\Big((1+e^2)\sin E_1 - \frac{3}{2} e E_1 - \frac{e}{4}\sin 2E_1 - (1+e^2)\sin E_0 + \frac{3}{2} e E_0$$

$$+ \frac{e}{4}\sin 2E_0\Big) R_{11} + \Big(\frac{e}{4}\sqrt{1-e^2}\cos 2E_1 - \sqrt{1-e^2}\cos E_1$$

$$- \frac{e}{4}\sqrt{1-e^2}\cos 2E_0 + \sqrt{1-e^2}\cos E_0\Big) \quad R_{12}\Bigg]$$

$$\frac{\partial R}{\partial e} = \frac{f_r}{2\pi}\Bigg[R_{11}\Big(-\frac{3}{2}E_1 + e\sin E_1 + \frac{1}{4}\sin 2E_1 + \frac{3}{2}E_0 - e\sin E_0 - \frac{1}{4}\sin 2E_0\Big)$$

$$+ R_{12}\Big(\frac{1}{\sqrt{1-e^2}}\Big(e\cos E_1 - \frac{1}{4}\cos 2E_1 - e\cos E_0 + \frac{1}{4}\cos 2E_0\Big)\Big)\Bigg]$$

$$\frac{\partial R}{\partial (N, I, w)} = \frac{f_r}{2\pi} \quad (1, 0, 0)\, \frac{\partial \underline{R}_{sq}}{\partial (N, I, w)} \left\{\begin{matrix} -\frac{3}{2} e E + (1+e^2)\sin E - \frac{e}{4}\sin 2E \\ \\ -\sqrt{1-e^2}\cos E + \frac{e}{4}\sqrt{1-e^2}\cos 2E \\ \\ 0 \end{matrix}\right\}_{E_0}^{E_1}$$

$$\frac{\partial R}{\partial M} = \frac{f_r a}{2\pi}\left[R_{11}(\cos E_1 - \cos E_0) + R_{12}\sqrt{1-e^2}(\sin E_1 - \sin E_0)\right],$$

where $\underline{R}_{sq}$ is a rotation matrix relating the position of the satellite in an orbital plane coordinate system to its position with respect to a coordinate system pointing at the sun. $\underline{R}_{sq}$ is defined as the successive rotations:

$$\underline{R}_3 (\lambda_{\odot}) \cdot \underline{R}_1 (\epsilon) \cdot \underline{R}_3 (-N) \cdot \underline{R}_1 (-I) \cdot \underline{R}_3 (-w),$$

where,

$$\underline{R}_1 (x) = \left\{ \begin{matrix} 1 & 0 & 0 \\ 0 & \cos x & \sin x \\ 0 & -\sin x & \cos x \end{matrix} \right\}, \text{ and}$$

$$\underline{R}_3 (x) = \left\{ \begin{matrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{matrix} \right\}.$$

R_{11} and R_{12} are elements of $\underline{R}_{sq}$.

Precession and Nutation

The effects of precession and nutation of the earth's polar axis are accounted for in ROAD by maintaining the integration in an inertial system with respect to the equinox and true equator at epoch (TE). All the forces are initially calculated in a system referred to the equinox and true equator of date (TD), corresponding to the observed data. The forces (actually the Kepler rates) are then rotated (by a precession-nutation matrix) to refer to the TE system where the integration proceeds. The integrated state is rotated back to the TD system at the end of each integration step for calculation of new rates. The chain of rotations actually used to obtain the rates in the TE system is:

$$\frac{d\,\bar{s}_{TE}}{d\,t} = \left(\frac{\partial\,\bar{s}_{TE}}{\partial\,\bar{x}_{TE}} \; \frac{\partial\,\bar{x}_{TE}}{\partial\,\bar{x}_{TD}} \; \frac{\partial\,\bar{x}_{TD}}{\partial\,\bar{s}_{TD}} \right) \frac{d\,\bar{s}_{TD}}{d\,t} \,,$$

where $\vec{x}$ is the cartesian position-velocity vector corresponding to the Kepler elements s in the indicated coordinate system. The matrix $\partial\bar{x}/\partial\bar{s}$ is documented on page 68 of Kaula's book [Kaula, 1966]. The precession-nutation matrix $\partial\bar{x}_{TE}/\partial\bar{x}_{TD}$ is described in volume I of the GEODYN Documentation, (Sept. 30, 1972, Goddard Space Flight Center NAS-5-11735 Mod. 65, PCN 550W-72416) and provides for Simon Newcomb's description of the precession and Edgar Woolard's nutation.

Solar and Lunar Tides

The ROAD program treats the tidal perturbations in a manner analogous to all other gravitational perturbations. The potential is expanded in terms of Kepler elements, and the long periodic terms only are selected for differentiation in the computation of the Kepler element rates from the standard form of the Lagrange equations (2). The form of the potential is due to Kaula [Kaula, 1969], and includes provision for latitude dependent tidal amplitudes (Love numbers) and phase lags (of the potential bulge from the 3rd body-earth line).

The potential that is used in ROAD is equation (22) in Kaula, 1969:

$$T = \sum_{\ell m p q h k j} K_{\ell m p q} \left(\frac{a_e}{a}\right)^{k+1} F_{kmj}(I)\, G_{kj(2j-k)}(e)\, Q_{\ell h k m}$$

$$\left[k_{\ell h} \left\{ \begin{matrix} \cos \\ \\ (-1)^m \sin \end{matrix} \right\}_{k\ \mathrm{odd}}^{k\ \mathrm{even}} + (k_\ell\, \epsilon)_h \left\{ \begin{matrix} -\sin \\ \\ (-1)^m \cos \end{matrix} \right\}_{k\ \mathrm{odd}}^{k\ \mathrm{even}} \right]$$

$$[v_{kmj(2j-k)} - v^*_{\ell m p q}],$$

The subscripted k's are the love number's and the ϵ's are the phase lags of the tidal potential. In the above potential, no short period effects (in the satellite's mean anomaly) are considered. Both K_{2mpq} and Q_{2mpq} terms (the dominant ones for a distant 3rd body; $\ell = 2$) are defined in tables II and I of Kaula's 1969 paper. The F and G functions are the usual ones for satellite orbits defined in Kaula's book [Kaula, 1966]. The latitude dependence of the love number k_ℓ is defined by

$$k_\ell = \sum_h k_{\ell h}\, P_{h0}\, (\sin \phi),$$

and the phase lag $\epsilon_{\ell m p q}$ by

$$\epsilon_{\ell m p q} = \sum_n \epsilon_n\, (\ell\ m\ p\ q)\, P_{n0}\, (\sin \phi)$$

where the P's are the ordinary Legendre polynomials. The quantity $(k_\ell \epsilon)_h$ is the sum

$$k_{\ell_h} \sum_{n} \epsilon_n Q_{knsm}.$$

The longitude arguments are:

$$V_{kmj(2j-k)} - v^*_{\ell mpq} = [(k - 2j)\, w + m\, N] - [(\ell - 2p)\, w^* + (\ell - 2p + q) M^* + m\, N^*],$$

where the starred elements belong to the 3rd body.

Element Secular Rates and Accelerations

The ROAD program also can include the effect of up to 5 arbitrary element rates and accelerations (derivatives) of the six initial Kepler elements $[\overline{E}]$ of each arc. Thus, the elements $\overline{E}$ at time t are expanded in a Taylor series in the arbitrary derivatives $\overline{E}^{[n]}$ at the epoch time t_0:

$$\overline{E}(t) = \sum_{n=0}^{5} \frac{(t - t_0)^n}{n!} \overline{E}^{[n]}(t_0)$$

In addition, the program can propagate a constant bias in each arc of semi-major axis observations, to satisfy the equation:

$$a\,(\text{OBSERVED}) = a\,(\text{CALCULATED}) + a\,(\text{BIAS}).$$

This bias is often needed to account for the different definitions of the mean semi-major axis between ROAD and the observations used.

ORBIT AND GEODETIC PARAMETER ESTIMATION

The logic of ROAD in its estimation (or differential correction) mode follows the (now universally used) scheme of partitioned normals [Kaula, 1966]. Parameters are separated into "arc" parameters; that is, parameters unique to each orbital arc, and common parameters, or parameters common to all orbit arcs. In a multi-arc solution with common parameters, the common parameter matrix is first solved and then the arc matrices are successively solved by back-substitution.

Of course parameter and orbit estimation requires partial derivatives. In ROAD these are obtained by numerically integrating variational equations. The equations of motion have the form

$$\frac{d\,s}{d\,t} = \epsilon\ f\ (a,\ e,\ i,\ N,\ w,\ M,\ t)$$

where s is an array Kepler elements, and ϵ is a parameter to be estimated. Thus formation of the variational equations is elementary. The rigorous variational equation for a parameter ϵ that is numerically integrated is:

$$\frac{\partial}{\partial \epsilon}\left[\left(\frac{d\,s}{d\,t}\right)\right] = \frac{d}{d\,t}\left(\frac{\partial\,s}{\partial\,\epsilon}\right) = f\ (a,\ e,\ i,\ N,\ w,\ M,\ t)$$

$$+\ \epsilon \sum_i \frac{\partial f}{\partial s_i}\ \frac{\partial s_i}{\partial \epsilon}$$

However, because of the small interaction of effects, the second term on the right hand side is ignored for radiation pressure, Earth tides, and even drag parameter estimation.

In parameter estimation the ROAD program utilizes a Baysian least squares scheme; i.e., a-priori information is required on all adjusted parameters. However the program accepts only a-priori standard deviations. No covariances are allowed (they are assumed to be zero). In ROAD, a-priori information is used to control the conditioning of a problem and also the amount by which parameters are allowed to adjust. But because the full covariances information is not given, this adjustment control is not always rigorous.

RESULTS AND CALIBRATIONS

The ROAD program has undergone extensive calibration on long arcs of simulated osculating element data in the course of its development [i.e., Wagner, Putney and Fisher, 1970 and Guion, Lynn and Lynn, 1970]. While these calibration results have generally been successful they point up the need for accurate osculating to mean element conversion when ROAD operates in its rapid mean element mode. Since the most accurate observed elements on real orbits are osculating (i.e.; derived by numerical integration from complete force models), the availability of a good converter is not just academic. The results presented here (on both real and simulated data) were obtained with the use of a combined analytic-numerical converter discussed in Douglas, Marsh and Mullins,

1972. Previous ROAD preprocessing of osculating elements [Wagner, 1972] removed short period effects using the analytic theory of Brouwer, 1959. The new method first removes analytically all short period effects due to the geopotential through (4, 4) [Kaula, 1966]. The remaining effects are averaged numerically over an orbit revolution with respect to a secularly precessing ellipse.

The satellites used for the real data calibration were GEOS-1, GEOS-2, and PEOLE. GEOS 1 has a near critical inclination, and so experiences rather large perturbations from odd zonal harmonics. The orbit is also slightly eccentric ($e = 0.07$) so that radiation pressure perturbations are important. Drag, although very small, is detectible.

GEOS-2 has a more nearly circular orbit than GEOS-1, and is thus relatively much less affected by radiation pressure. However, the high (106°) inclination of the orbit causes a slow node rate (0°.6/day) and resulting large tidal perturbations exceeding 10 arc seconds in the inclination and 30 arc seconds in the node.

PEOLE is a low inclination (15°), low perigee satellite with rapid node and perigee rates. Drag is very high, the semi-major axis decaying about 30 m per day. All of these satellites are significantly perturbed by the sun, moon, and the effects of precession and nutation.

The orbit of PEOPLE is a good test of the ability of ROAD to represent properly the effects of atmospheric drag. Figure 1 shows the observed and computed semi-major axes of PEOLE obtained from a ROAD orbit determination. The time covered is the year 1971. The mean elements were determined from 4 day orbital arcs of minitrack and laser data as part of the International Satellite Geodesy Experiment (ISAGEX) [Marsh, Douglas and Klosko, 1971]. Note the close agreement of observed and computed elements using the Jacchia 1968 model atmosphere. Subtle variations are visible and are properly modeled.* The overall error for the year is only about 5%, and appears to be long-periodic. It is suspected that the Jacchia atmosphere of 1968 may have an error in the semi-annual variation of density, so a small unmodeled variation is not surprising (see also Wagner, 1972).

Figure 2 shows the mean eccentricity of PEOLE, also obtained from the ISAGEX data. The periodic variation due to the odd zonal harmonics is clearly visible, as is the secular decrease due to drag. Figure 3 shows the residual eccentricity with drag modeled as above and zonal harmonics from the SAO 1969 Standard Earth [Gaposchkin and Lambeck, 1970]. Note that drag effects are removed essentially perfectly (the residuals have no clear secular trends) but a residual odd zonal harmonic effect remains. Since the SAO 1969 zonal harmonics included no low inclination satellites, such a residual effect is to be anticipated. To verify that this is not due to error in ROAD, the eccentricity of PEOLE was obtained from a complete "Cowell"-type, numerical integration of the geopotential to (4, 4). Mean elements were then prepared from

* Where observed and computed points are coincident, only the observed is printed.

this trajectory by the method in Douglas, Marsh, and Mullins (1972). Figure 4 shows the variation in eccentricity produced by $C_{3,0}$ (predominantly) and Figure 5 shows the ROAD fit to this data using the same value of $C_{3,0}$. The residuals appear random and have an rms of only 0.1% of the amplitude of the eccentricity variation shown in Figure 4.

GEOS-1 offers a good opportunity to study the ROAD radiation pressure model. Drag is small for GEOS-1 (but not negligible). Resonance produces a small, significant effect, but with a period of less than a week. Because of the substantial eccentricity of the orbit, radiation pressure causes the dominant perturbation of the semi-major axis during orbit shadow periods. Fluctuations of 5-15 meters over several months are typical. These can be studied in detail because very accurate mean elements for GEOS-1 are available. The mean elements obtained in Douglas, Marsh, and Mullins (1972) from 2 day orbital arcs of optical data have a precision of 25 cm in the semi-major axis (see figure 6).

The decay of semi-major axis of GEOS 1, due to drag, amounts to 5-10 m per year and is most readily seen during a no-shadow period, such as July-August 1966. Because of these significant drag effects, it was found necessary to account for both drag and radiation pressure perturbations on GEOS-1.

The radiation pressure model in ROAD does not include the Earth Albedo effect. Thus a small error in the modeling of radiation pressure is possible. The period March 11 - May 15, 1966 on GEOS-1 has been studied intensively for determination of tidal parameters where, of course, accurate modeling of radiation pressure is essential. During this period the semi-major axis is perturbed in a non-linear fashion by radiation pressure by about 5-6 meters, and in a linear way from drag by about 60 cm. Figure 7 shows the residuals obtained in the mean semi-major axis for this period by ROAD when C_D and C_R are adjusted to the values $C_D = 3.1$, and $C_R = 1.52$. The rms of the fit is less than 25 cm.

Another important test of ROAD concerns Earth tides. Figure 8 shows the residuals in the inclination of GEOS 2 over a 600 day period with respect to mean elements obtained from 2 day arcs of optical tracking data. Earth tides were not modeled in obtaining these residuals. Note a long period unmodeled perturbation of amplitude 5" in the inclination. Figure 9 shows the results with the Love number $k_2 = 0.30$ modeled in ROAD. The residuals are almost random. This value of k_2 also substantially explains a similar, substantial, solar tidal effect (of the same long period) in the node of GEOS 2. This value of the Love number is in reasonable agreement with values derived from other orbits which range from 0.25 to 0.35. [Kaula, 1969].

SUMMARY

The ROAD program can efficiently analyze long arcs of mean elements for a wide variety of geodetic effects. Since 1969, the program has been used extensively to determine and evaluate both resonant and zonal geopotential harmonics from a large number of orbits. It has also been used to investigate possible variations in low order earth-gravity "constants" [Wagner, 1973].

Currently, the program is also being used at Goddard Space Flight Center to study the time varying geopotential due to sun and moon induced earth tides, through their effects on a large number of well tracked satellite orbits [Douglas and Marsh, 1973].

REFERENCES

Brouwer, D., Solution of the problem of artificial satellite theory without drag, Astronomical Journal, 64, No. 1274, 378-397, Nov., 1959.

Diamante, J. M. and V. Der, Upper atmosphere density satellite drag models, 58 pp., Wolf Research and Development Corp. (Under NASA Contract NAS 5-11735 Mod 60 and 65), Riverdale, Md., Sept., 1972.

Douglas, B. C., P. J. Dunn and R. G. Williamson, The ROAD program, 41 pp., Wolf Research and Development Corp. (Under NASA Contract NAS 5-11736-163), Riverdale, Md., 1972.

Douglas, B. C., J. G. Marsh and N. E. Mullins, Mean elements of GEOS 1 and GEOS 2, Goddard Space Flight Center Document X-553-72-85, Greenbelt, Md., March, 1972.

Douglas, B. C., S. M. Klosko, J. G. Marsh and R. G. Williamson, Tidal perturbations on the orbit of GEOS I and GEOS II, Wolf Research and Development Corp., Riverdale, Md., 1973.

Gaposchkin, E. M., and K. Lambeck, 1959 Smithsonian standard earth II, Smithsonian Astrophysical Observatory Special Report 315, p. 8, 70, Cambridge, Mass., May, 1970.

Gedeon, G. S., B. C. Douglas and M. T. Palmiter, Resonance effects on eccentric satellite orbits, The Journal of the Astronautical Sciences, 14, #4, 147-157, July-August, 1967.

Guion, S. E., D. L. Lynn and J. J. Lynn, Final report for ROAD-2, CAMEL and REGRAP programs, 1-42, DBA Systems Inc. (Under NASA Contract NAS 5-11730), 9301 Annapolis Road, Lanham, Md., Sept., 1970.

Kaula, W. M., Development of the lunar and solar disturbing functions for a close satellite, Astronomical Journal, 67, No. 5 300-303, June, 1970.

Kaula, W. M., Theory of Satellite Geodesy, 124 pp., Blaisdell Publishing Co., Waltham, Mass., 1966.

Kaula, W. M., Tidal friction with latitude-dependent amplitude and phase angle, Astronomical Journal 74, No. 9, 1108-1114, Nov. 1969.

King-Hele, D., Theory of Satellite Orbits in an Atmosphere, 165 pp., Butterworths Press, London, England, 1964.

Kozai, Y., The motion of a close earth satellite, Astronomical Journal, 64 No. 1274, 371, Nov. 1959.

Marsh, J. G., B. C. Douglas and S. M. Klosko, Geodetic analyses from the international satellite geodesy experiment (ISAGEX) data, Goddard Space Flight Center, Greenbelt, Md., May, 1972 (paper presented at the International Symposium on Satellite and Terrestrial Triangulation, Graz, Austria, May 29 - June 2, 1972).

Wagner, C. A., Preliminary results from mean element analysis of 12-hour resonant orbits, Goddard Space Flight Center Document X-552-69-264, 1-7, Greenbelt, Md., June, 1969

Wagner, C. A. and B. C. Douglas, Resonant satellite geodesy by high speed analysis of mean kepler elements, in: Dynamics of Satellites (1969), 130-137, Springer-Verlag, Pub., Berlin, West Germany, 1970.

Wagner, C. A., Geopotential coefficient recovery from very long arcs of resonant orbits, Journal of Geophysical Research, 75, No. 32, 6662-6674, Nov. 10, 1970b.

Wagner, C. A., B. H. Putney and E. R. Fisher, Recovery of zonal geopotential information from goddard minitrack data, Goddard Space Flight Center Document X-552-70-160, 25 pp., Greenbelt, Md., May, 1970.

Wagner, C. A., Low degree resonant geopotential coefficients from eight 24-hour satellites, Goddard Space Flight Center Document X-552-70-402, 40 pp., Greenbelt, Md., Oct., 1970a.

Wagner, C. A., Earth zonal harmonics from rapid numerical analysis of long satellite arcs, Goddard Space Flight Center Document X-553-72-341, 32 pp., Greenbelt, Md., August, 1972.

Wagner, C. A., Does $\lambda_{2,2}$ vary? Journal of Geophysical Research, Jan. 10, 1973.

Wexler, D. M. and G. S. Gedeon, Rapid orbit prediction program (ROPP), TRW Systems Ground Document 08554-6001-R000, 1-87, Redondo Beach, California, December 5, 1967.

Williamson, R. W., ROAD program description and user's guide, 26 pp., Wolf Research and Development Corp. (Under NASA Contract NAS 5-9756, 182), Riverdale, Md., 1970.

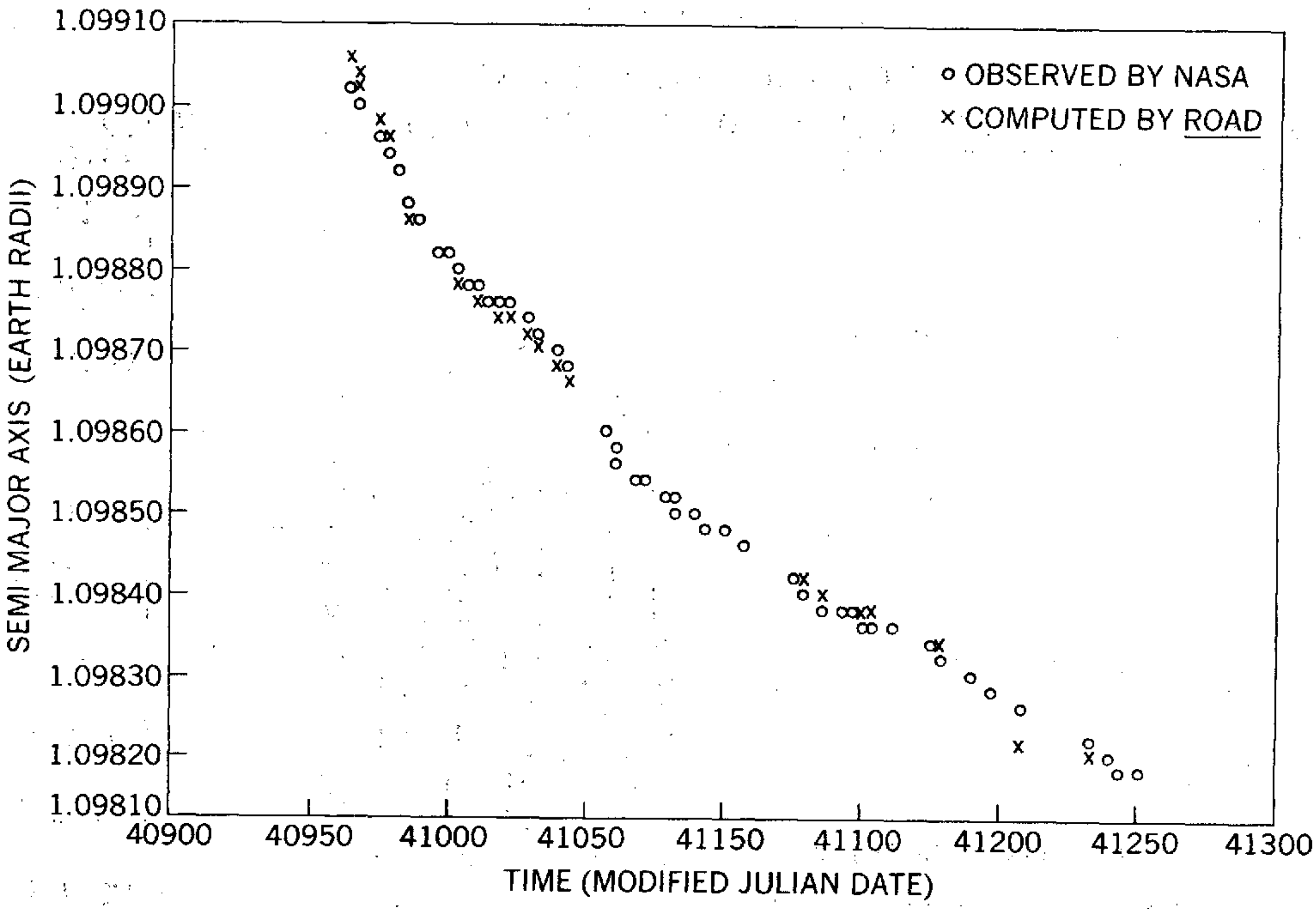

Figure 1. The Semi-major Axes of PEOLE Obtained From a Road Orbit Determination.

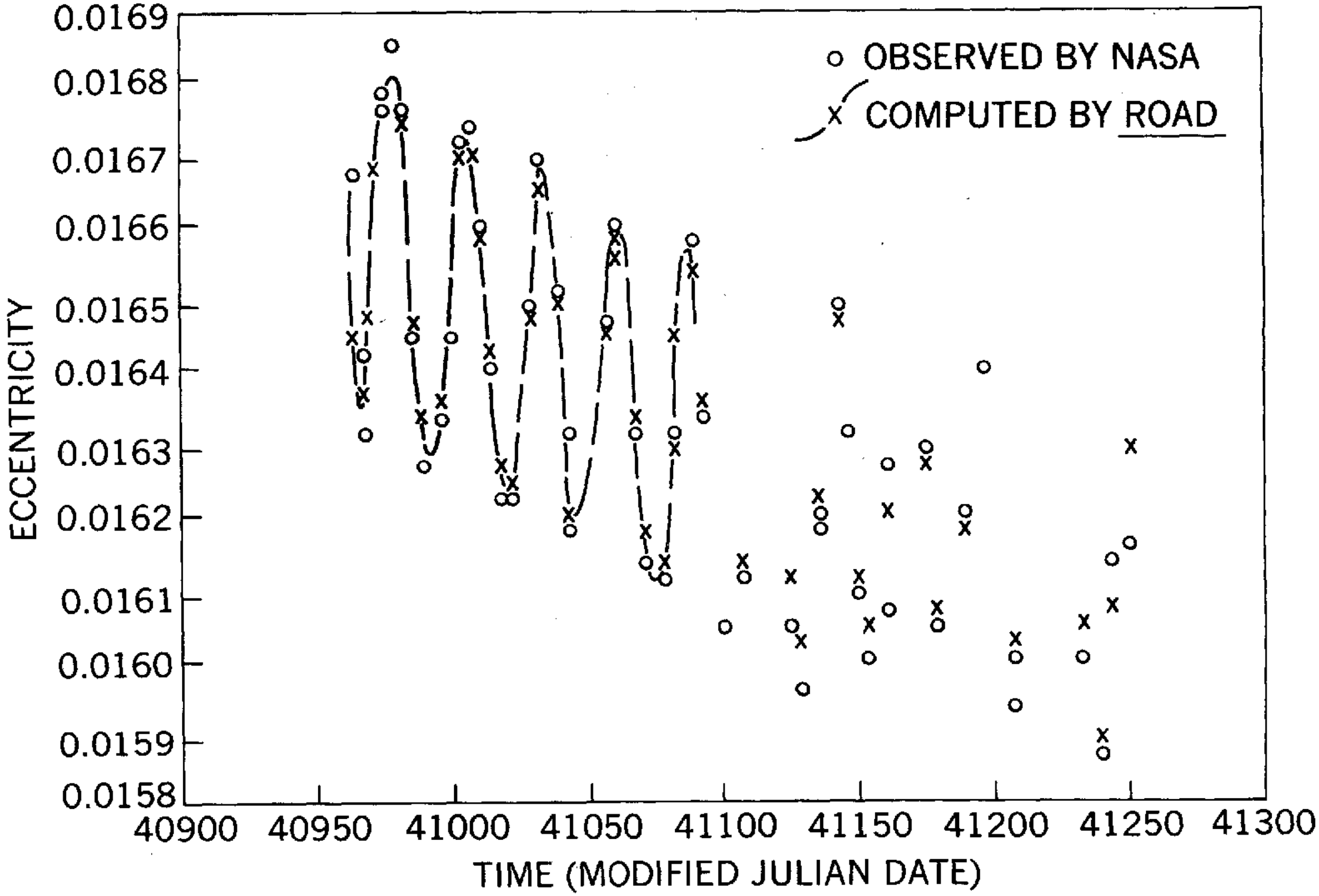

Figure 2. The Eccentricity of PEOLE.

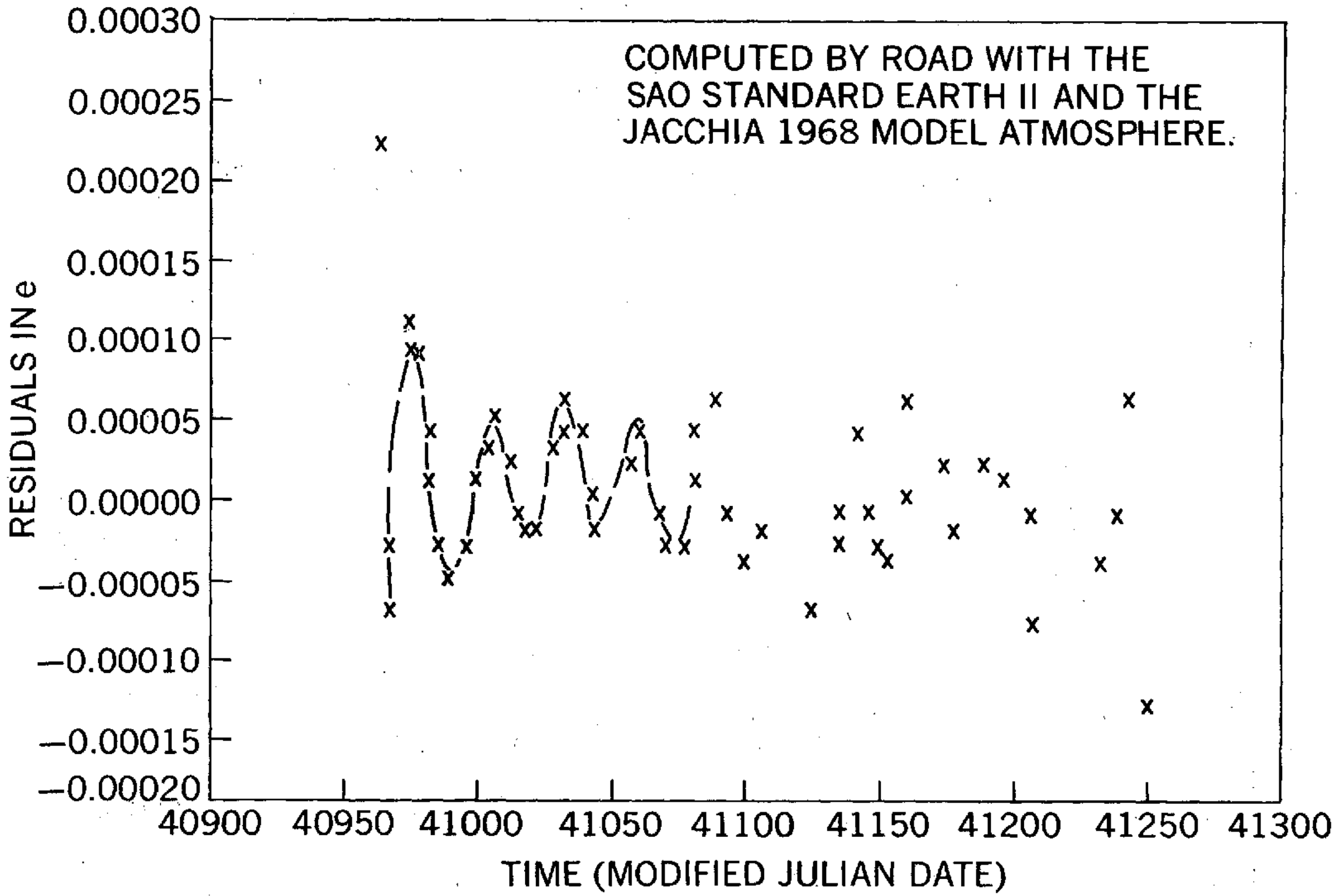

Figure 3. Eccentricity Residuals for PEOLE.

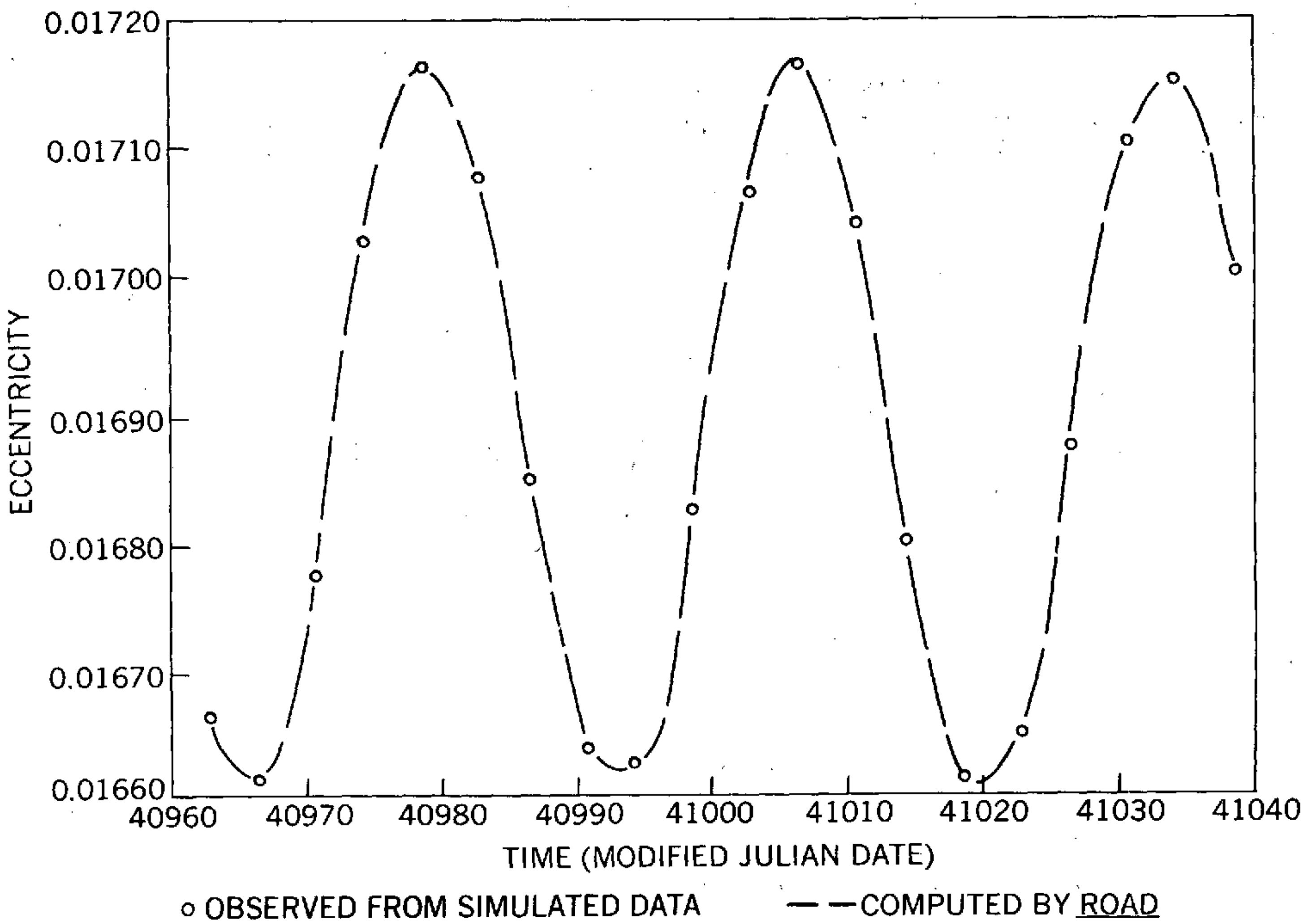

Figure 4. The Mean Eccentricity From a Simulated PEOLE Trajectory.

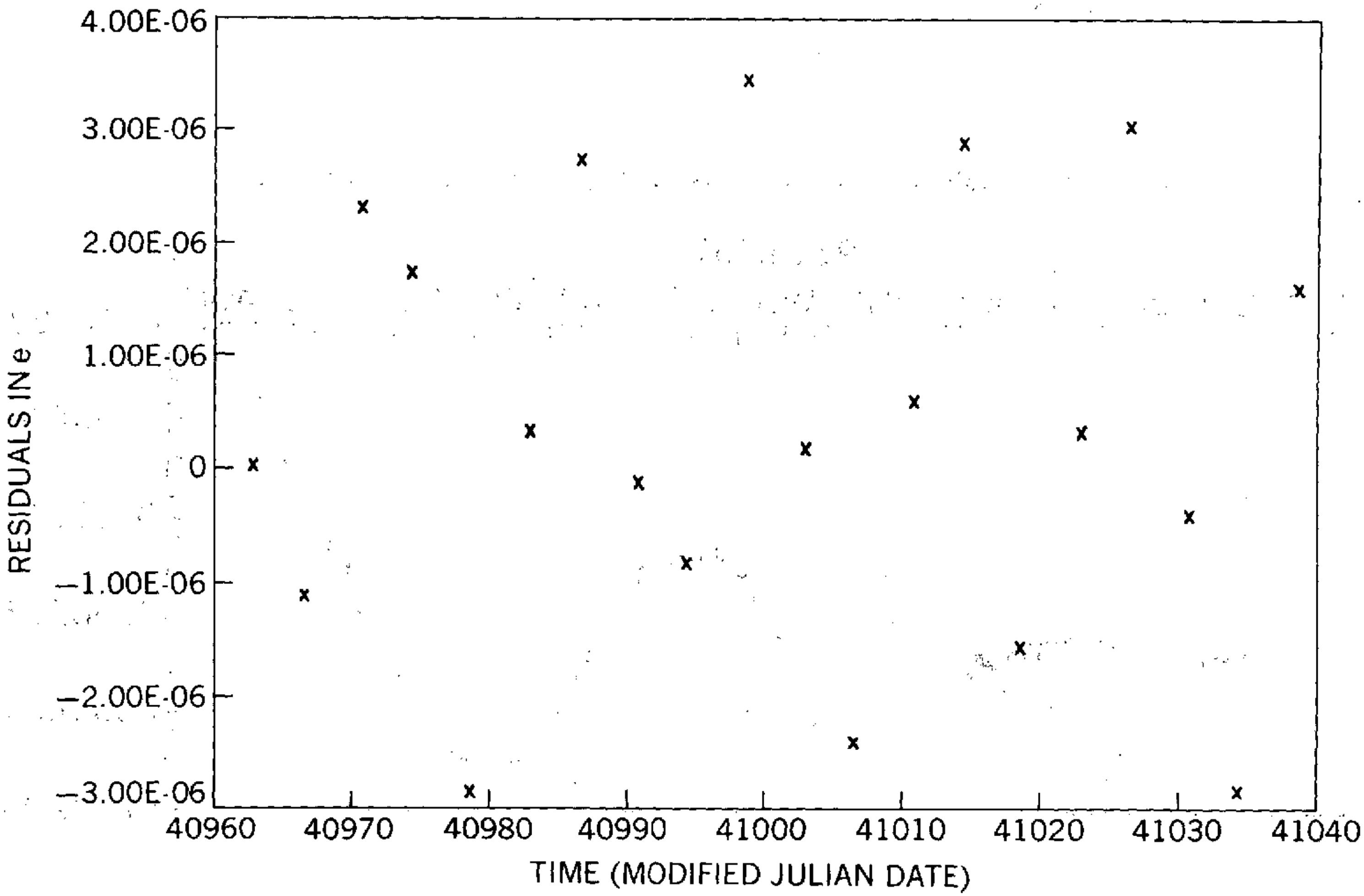

Figure 5. Residuals in Eccentricity Computed by ROAD From a Simulated PEOLE Trajectory.

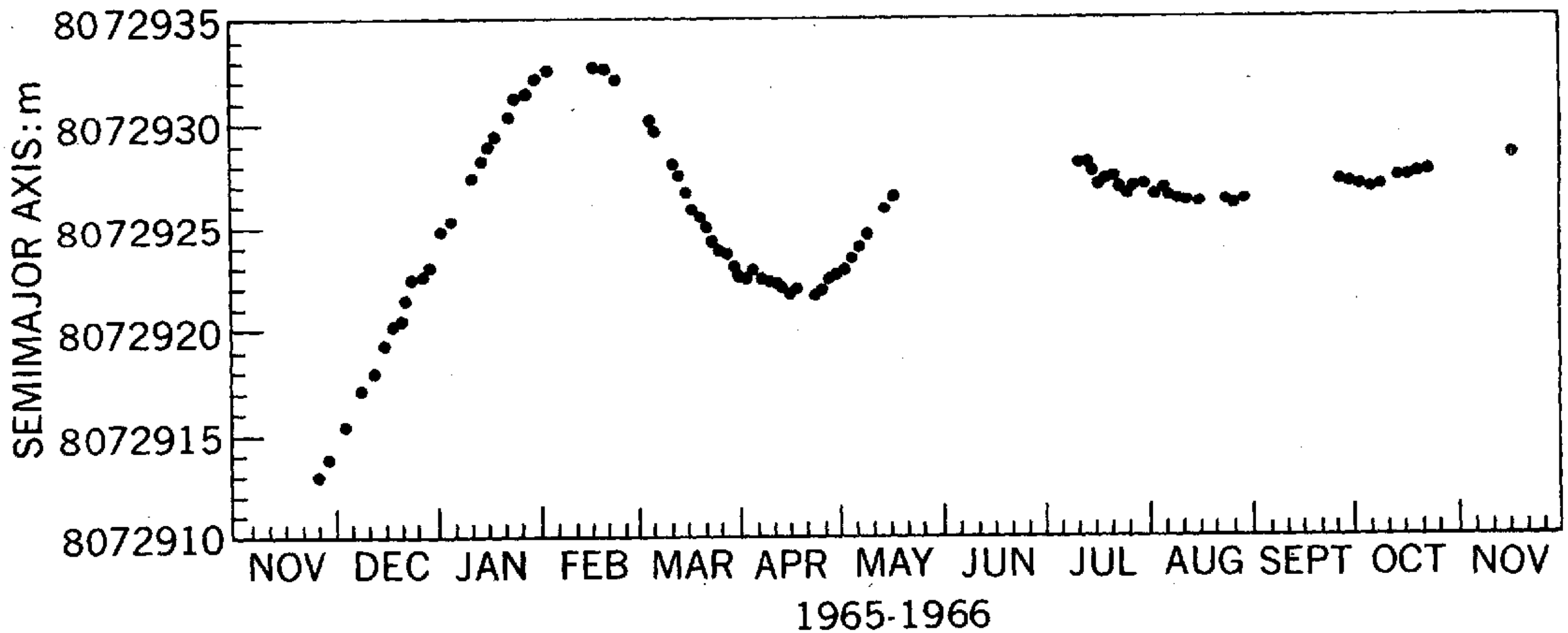

Figure 6. GEOS 1 Mean Semi-Major Axis from Two-Day Optical Data Arcs.

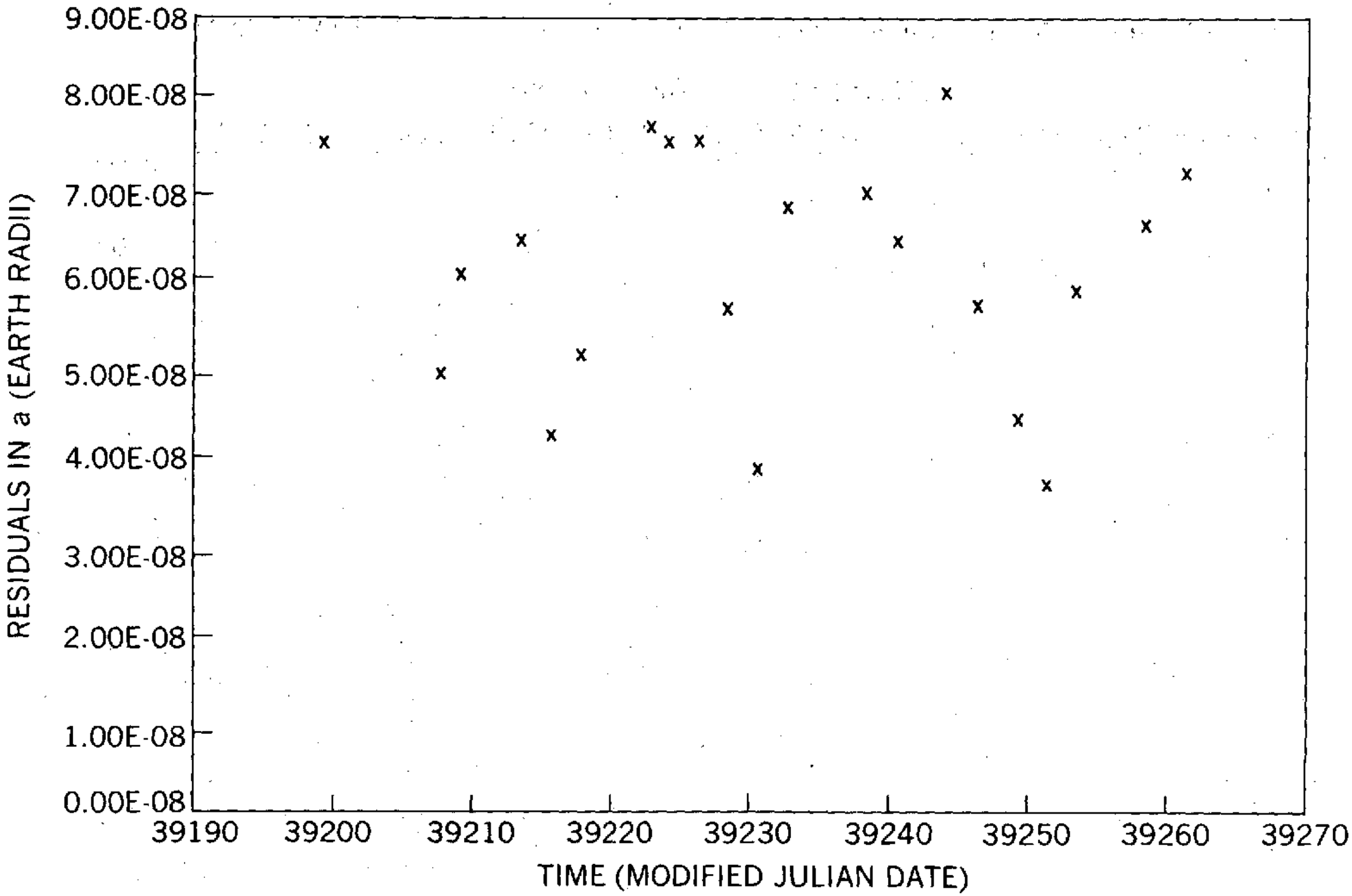

Figure 7. Residuals in the Semi-Major Axis from a ROAD Orbit Determination of GEOS-1 Adjusting C_R and C_D.

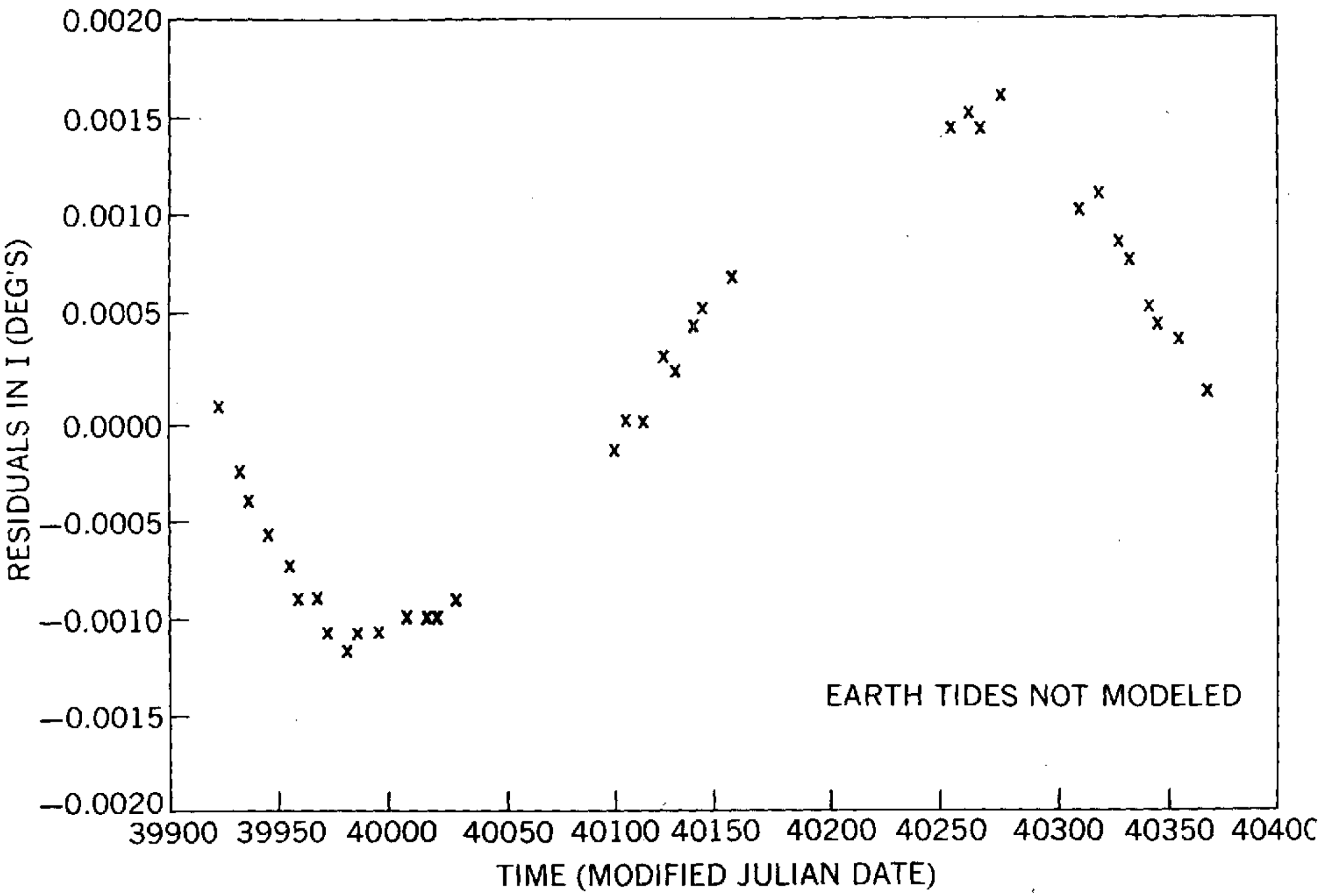

Figure 8. Residuals in the Inclination of GEOS-2 from a ROAD Orbit Determination

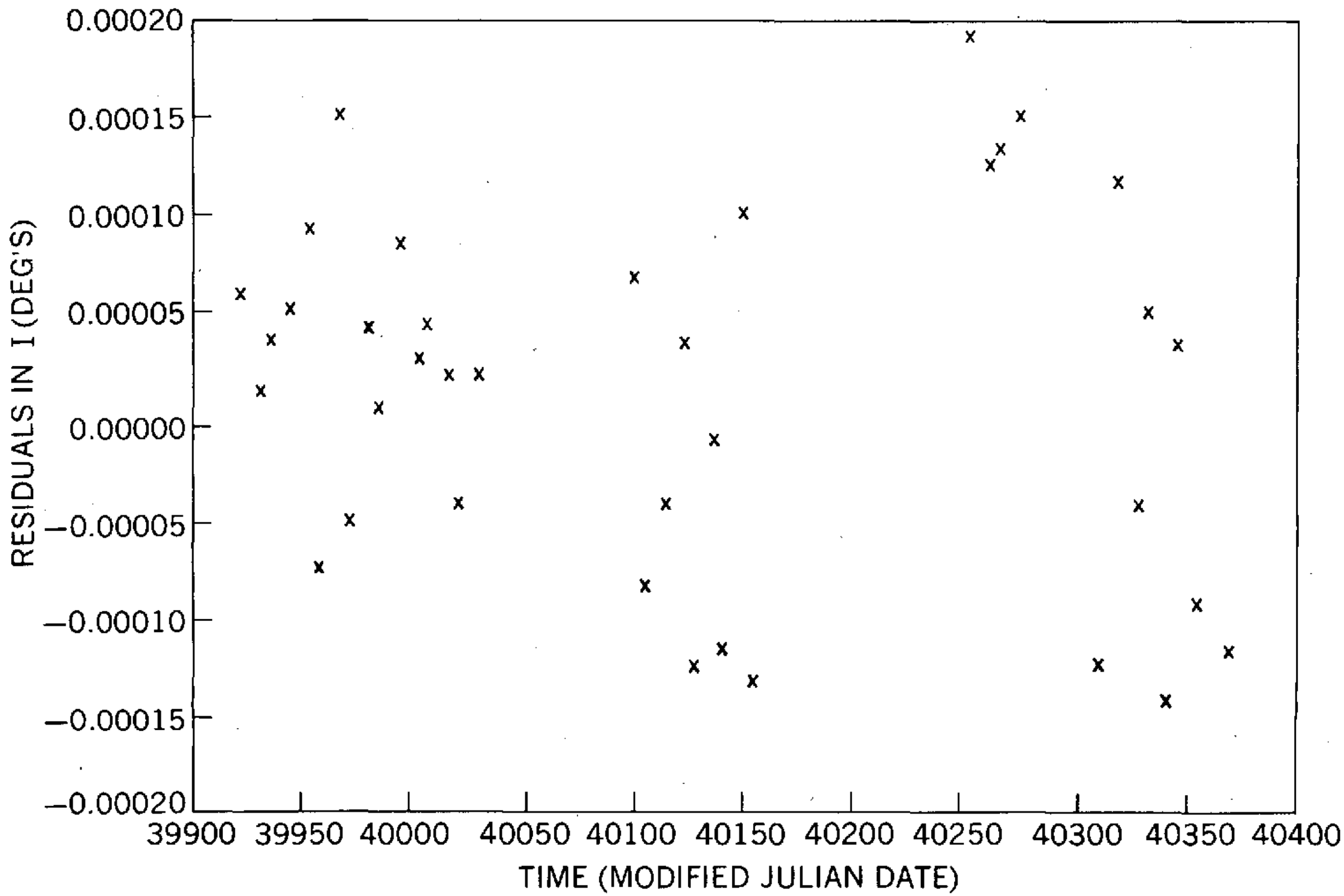

Figure 9. Residuals in the Inclination of GEOS-2 Modeling Tides at $K_2 = .30$.